Passive and Static Localization of an Electromagnetic Source using Interval Methods

Guillaume Garde, Lionel Lapierre and Luc Jaulin

ENSTA - Institut Polytechnique de Paris - Lab-STICC UMR 6285
2 rue François Verny, Brest, France
{guillaume.garde,lionel.lapierre,luc.jaulin}@ensta.fr

Keywords: Passive Localization, Swarm, Interval Methods

Introduction

Source localization is a key concept in various domains that exploit signal analysis techniques, such as telecommunication, astrophysics, robotics, or electronic warfare (EW) [4]. The latter aims to master the electromagnetic (EM) spectrum in defense contexts in order to maximize detection capabilities while remaining undetected [6]. Hence the need for efficient passive localization methods—that is, using incoming electromagnetic signals, such as RADAR signals, to localize the source without emitting anything to ensure stealth. Passive localization methods of EW have become quite accurate, but the corresponding equations are solved using estimators [9]. This means there is an associated uncertainty and a risk of losing valid solutions. A new approach using the same equations with interval methods is proposed here. It shows that it is possible to localize a source with guaranteed bounds using well-known passive localization principles.

Properties and Method

A peculiar attention is given to three different techniques. The Time Difference Of Arrival (TDOA) [7] [9], the Direction Of Arrival (DOA)

[5], and, for the specific context of scan-based emitters, the Time Difference Of Arrival Between Lobes (TDOA-BL) [1]. These methods can be combined to gain performance and accuracy depending on the scenario. Some hypotheses are made for this work. The emitter and the observers are considered to be static and to live in a two dimension plan. The problem of synchronization between observers is not addressed here. Future work will enable the use of more general assumptions.

Notation

Consider a two dimension cartesian plan. Let $P(x, y)$ be the emitter of unknown position. A network of n obervers is represented by points $O_i(x_i, y_i), 0 \leq i \leq n$. Let c denote the speed of light. For an observer O_i, the time of arrival of a signal is t_i and the angle of arrival with repect to the North is α_i. In case of a scan-based rotating emitter, the rotation speed of the antenna is called w and is supposed to be constant.

TDOA

For a pair of observers (O_i, O_j) receiving the same EM signal the time difference of arrival equation is

$$t_i - t_j = \frac{1}{c}(\sqrt{(x - x_i)^2 + (y - y_i)^2} - \sqrt{(x - x_j)^2 + (y - y_j)^2}).$$

The solution of this equation is an hyperbola [2][9].

DOA

For an observer O_i receiving an EM signal, the direction of arrival equation is

$$\theta_i = \text{atan2}(y - y_i, x - x_i).$$

This equation gives a line passing through the source and the observer [5].

TDOA-BL

For a pair of observers (O_i, O_j) crossed through by a rotating EM beam, the time difference of arrival between lobes equation is

$$\alpha_i - \alpha_j = \mathrm{atan2}(y - y_i, x - x_i) - \mathrm{atan2}(y - y_j, x - x_j).$$

And

$$t_i - t_j = \frac{\alpha_i - \alpha_j}{\omega}$$

The solution is a circle arc [1].

Solving

In this work, these equations are solved using the SIVIA algorithm [3] for set inversion, and specific contractors and separators have been implemented using the CODAC library [8]. As the number of observers is increased, the solutions are given by the intersection of multiple sets. Different scenarios have been considered and played via a simulator. As sensors are not ideal, each measure or position is modeled by an interval whose length depends on the chosen uncertainty.

Main results

We provide here some computational results. Observers are represented by black boxes and the EM source by a red one. These boxes have been voluntarily enlarged for the purpose of representation. The solution sets are approached by an outside part (in blue), an inside part (in green), and bound (in orange).

Figures 1, 2 and 3 show the computed results of respectively a 3 observers TDOA scenario, a 3 observers DOA scenario and a 3 observers TDOA-BL scenario.

These methods are highly geometrical. Interval methods are particularly well suited for this. The major interest of this work is the

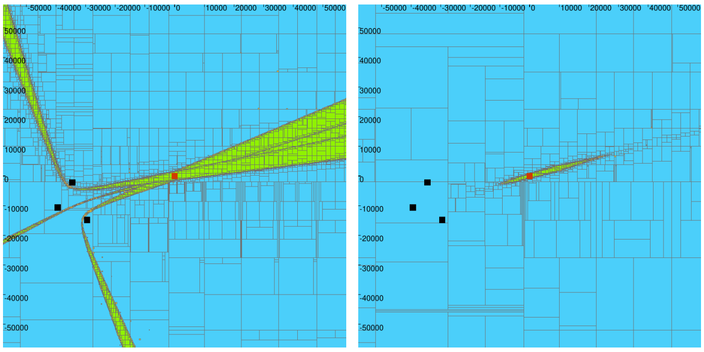

Figure 1: TDOA scenario

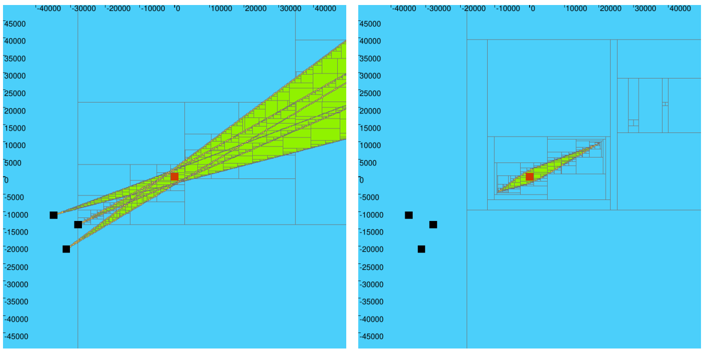

Figure 2: DOA scenario

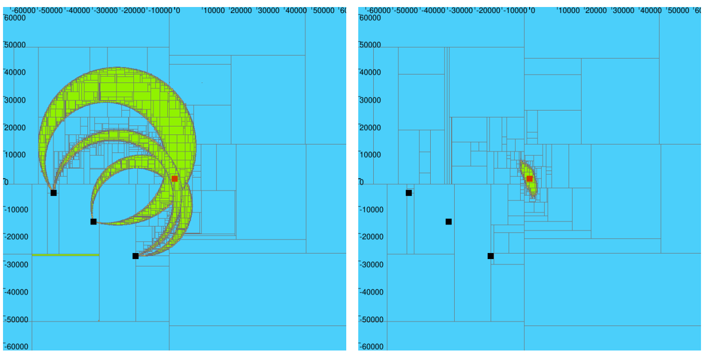

Figure 3: TDOA-BL scenario

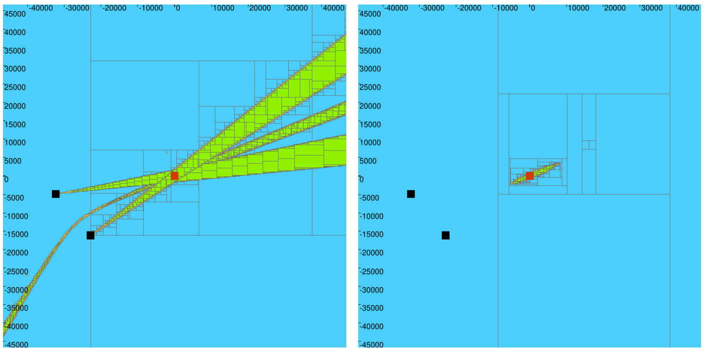

Figure 4: TDOA and DOA mixed scenario

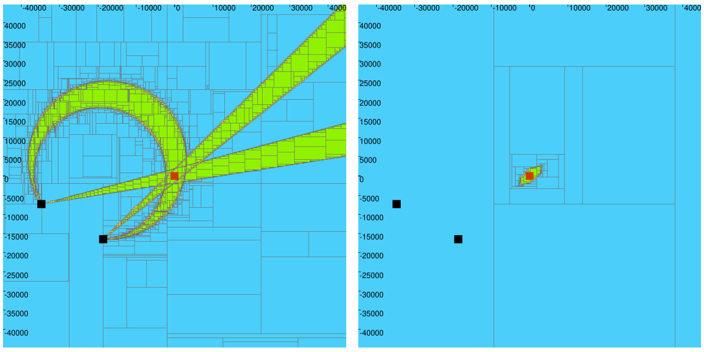

Figure 5: TDOA-BL and DOA mixed scenario

possibility of mixing them. Figures 4 and 5 show the computed results of 2 scenarios mixing first TDOA and DOA, then TDOA-BL and DOA. Only 2 observers are needed to obtain these results.

References

[1] H. Hmam. Scan-based emitter passive localization. *IEEE Transactions on Aerospace and Electronic Systems*, 43(1):36–54, 2007.

[2] Luc Jaulin. Optimal separator for an hyperbola; application to localization, 2023.

[3] Luc Jaulin and Eric Walter. Set inversion via interval analysis for nonlinear bounded-error estimation. *Automatica*, 29(4):1053–1064,

1993.

[4] C. Knapp and G. Carter. The generalized correlation method for estimation of time delay. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, 24(4):320–327, 1976.

[5] Alien G. Lingren and Kai F. Gong. Position and velocity estimation via bearing observations. *IEEE Transactions on Aerospace and Electronic Systems*, AES-14(4):564–577, 1978.

[6] Reeshen Reddy and Saurabh Sinha. State-of-the-art review: Electronic warfare against radar systems. *IEEE Access*, 13:57530–57567, 2025.

[7] O. Reynet, L. Jaulin, and G. Chabert. Robust tdoa passive location using interval analysis and contractor programming. In *2009 International Radar Conference "Surveillance for a Safer World" (RADAR 2009)*, pages 1–6, 2009.

[8] Simon Rohou, Benoît Desrochers, and Fabrice Le Bars. The codac library: A catalog of domains and contractors. *Acta Cybernetica*, 26(4):871–887, Mar. 2024.

[9] Hugo Seuté, Cyrille Enderli, Jean-François Grandin, Ali Khenchaf, and Jean-Christophe Cexus. Experimental measurement of time difference of arrival. In *2016 17th International Radar Symposium (IRS)*, pages 1–4, 2016.