

Implicit Taylor Series Method for Delay Differential Equations

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We present an implicit Taylor method algorithm for rigorous integration of delay differential equations. It is a modification to algorithms considered in [1] and [2] which use explicit Taylor method to compute rigorous bounds on a set of solutions to delay differential equations of the form

$$\begin{cases} x'(t) = f(x(t), x(t - \tau_1), \dots, x(t - \tau_m)) & t \geq 0 \\ x(t) = \psi(t) & t \in [-\tau, 0] \end{cases}$$

where $\tau = \tau_1 \leq \dots \leq \tau_m \leq 0$.

We extend them by additionally using implicit Taylor method together with interval Newton method to further improve the bounds on the solutions. It is especially important when studying stiff equations, where the explicit method might fail to converge and the errors can accumulate quickly.

References

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