

A $(max, +)$ based interval contractor for swarm localization

Luc Jaulin¹ and Laurent Hardouin²

¹ ENSTA, Lab-STICC
2 rue François Verny, 29200, Brest
luc.jaulin@ensta.fr

² Polytech Angers, LARIS
62 avenue notre-Dame du Lac, 49000 Angers
laurent.hardouin@univ-angers.fr

Keywords: Max plus algebra, Localization, Interval Methods

In this work, we propose an interval based approach [4] to improve the accuracy of the global localization and drastically reduces the uncertainties of the mutual positioning using the principle of DGPS (Differential Global Positioning Systems). Uncertainties will be represented by intervals.

Consider m robots $\mathcal{R}_i$, $i \in \{1, 2, \dots, m\}$, described by the following state equations:

$$\begin{pmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \\ \dot{v}_i \end{pmatrix} = \begin{pmatrix} v_i \cos \theta_i \\ v_i \sin \theta_i \\ \omega_i \\ \alpha_i \end{pmatrix} \quad (1)$$

where (x_i, y_i) is the position of $\mathcal{R}_i$, θ_i is its heading and v_i is the speed. The inputs are the yaw ω_i and the acceleration α_i .

We assume that for each time $t_k = k \cdot dt_k$, $k \in \{1, 2, 3, \dots\}$,

- each robot $\mathcal{R}_i$ measures its position $(x_i(k), y_i(k))$ with a precision $\pm e_{GPS}$ for each component. This means that for each t_k , we have $2m$ intervals $[x_i]$ and $[y_i]$ such that $x_i \in [x_i]$ and $y_i \in [y_i]$.
- for any pair of robots (i, j) , the vector

$$\begin{pmatrix} a_{ij}(k) \\ b_{ij}(k) \end{pmatrix} = \begin{pmatrix} x_j(k) - x_i(k) \\ y_j(k) - y_i(k) \end{pmatrix} \quad (2)$$

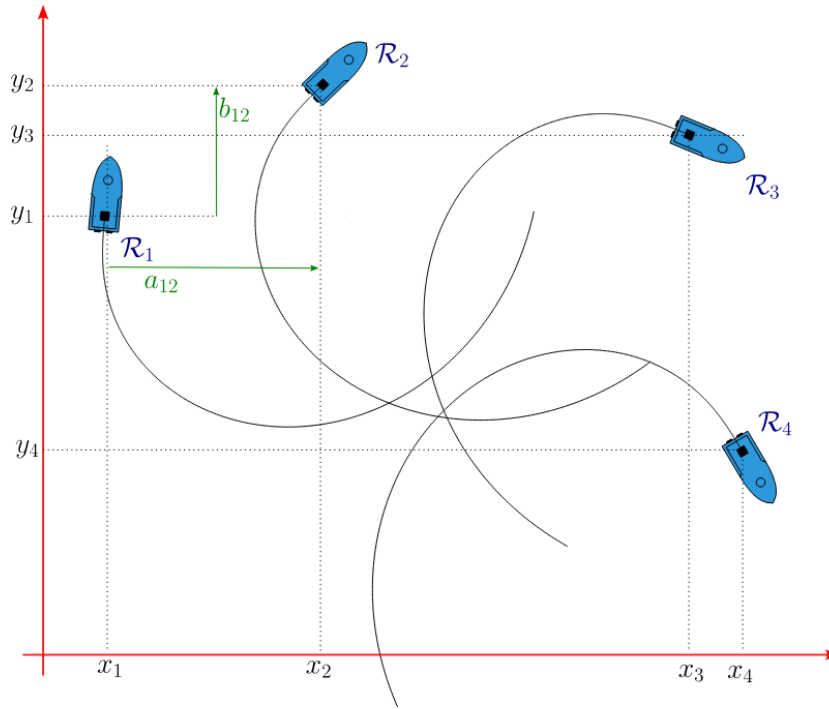
is measured with an error of $\pm e_{DGPS}$ for each component a_{ij}, b_{ij} . This means that for every t_k , we have $2m^2$ intervals $[a_{ij}]$ and $[b_{ij}]$ such that $a_{ij} \in [a_{ij}]$ and $b_{ij} \in [b_{ij}]$.

In our GNSS context, the size of the intervals $[a_{ij}], [b_{ij}]$ is small compared to the size of the intervals $[x_i]$ and $[y_i]$.

The figure below shows a situation with four mobile robots $\mathcal{R}_1, \dots, \mathcal{R}_4$. For times t_k , the m positions x_i, y_i and the m^2 mutual distances a_{ij}, b_{ij} are measured with some bounded uncertainties. Note that for each (i, j, ℓ) ,

$$\begin{aligned} a_{ij} &= a_{i\ell} + a_{\ell j} & (\text{Chasles}) \\ b_{ij} &= b_{i\ell} + b_{\ell j} \\ a_{ij} + a_{ji} &= 0 & (\text{symmetry}) \\ b_{ij} + b_{ji} &= 0 \end{aligned} \quad (3)$$

These relations can be used to contract the intervals for a_{ij}, b_{ij} . For this, we can take classical interval contractors [3], but the process would be slow due to the number of constraints that have to be considered (for Chasles: m^3 constraints for each t_k) and the propagation could be long. We would like to do better considering all these constraints as a global constraint and build a dedicated contractor to add to the catalog of global contractors [2]. In this work, we build this dedicated contractor using $(\max, +)$ algebra [1].



References

- [1] F. Baccelli, G. Cohen, G.J. Olsder, and J.P. Quadrat. *Synchronization and Linearity. An Algebra for Discrete Event Systems*. John Wiley & Sons, New York, 1992.
- [2] N. Beldiceanu, M. Carlsson, and J.X. Rampon. *Global Constraint Catalog*, available at www.emn.fr/x-info/sdemasse/gccat/. University of Nantes, 2009.
- [3] L. Jaulin, M. Kieffer, O. Didrit, and E. Walter. *Applied Interval Analysis, with Examples in Parameter and State Estimation, Robust Control and Robotics*. Springer-Verlag, London, 2001.
- [4] R. Moore. *Methods and Applications of Interval Analysis*. Society for Industrial and Applied Mathematics, jan 1979.