

Trajectory-based search for periodic orbits in a memristive circuit and verification of their existence using interval methods

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Introduction

Periodic orbits are important objects in the analysis of chaotic dynamical systems, since they describe recurrent structures embedded in a chaotic attractor. In this work we study periodic orbits of the Poincaré return map associated with a nonlinear memristive circuit exhibiting chaotic dynamics. The circuit model is based on the double locally active memristor-based inductor-free chaotic circuit proposed in [1].

The aim of the work is to combine a trajectory-based numerical search with interval methods. The numerical part is used to locate candidates for periodic orbits, while the rigorous part is used to verify the existence of selected orbits by means of an interval Newton method. The general strategy follows the idea that a long trajectory of a chaotic system can be used to detect approximate recurrent behaviour and to generate good initial guesses for Newton-type refinement [2].

Trajectory-based search for periodic orbits

The continuous-time dynamics is reduced to a discrete-time problem by introducing a Poincaré section

$$\Sigma = \{(x_1, v_C, x_2) \in \mathbb{R}^3 : x_1 = x_1^*\}.$$

The corresponding Poincaré return map is denoted by P . In the numerical experiments a long trajectory of the return map is generated and the consecutive return points

$$z_0, z_1, z_2, \dots$$

on the section are collected.

For a prescribed period p , candidate periodic points are detected by searching for close returns along this trajectory. More precisely, points satisfying

$$\|z_{k+p} - z_k\| \leq \delta$$

for a chosen tolerance $\delta > 0$ are treated as approximate period- p returns. Such close returns provide initial guesses for Newton's method applied to the equation

$$F_p(z) = P^p(z) - z = 0.$$

A zero of F_p corresponds to a fixed point of P^p , and therefore to a periodic orbit of the return map.

After Newton refinement, the obtained candidates are filtered. First, duplicate candidates are removed, since the same orbit may be found several times from different close returns along the same long trajectory. Second, candidates corresponding to shorter periods are eliminated. This gives a list of numerically refined periodic orbits with prescribed minimal periods.

Interval verification

The second stage of the procedure is devoted to rigorous verification. For each Newton-refined candidate $\bar{z}$, a small interval box $[z]$ centered

at $\bar{z}$ is constructed on the Poincaré section. Then an interval Newton operator is applied to the equation

$$F_p(z) = P^p(z) - z = 0.$$

In the computations this has the form

$$N([z], \bar{z}) = \bar{z} - [DF_p([z])]^{-1}F_p(\bar{z}),$$

where $[DF_p([z])]$ denotes an interval enclosure of the derivative of F_p over the box $[z]$. If

$$N([z], \bar{z}) \subset [z],$$

then the existence of a true zero of F_p in $[z]$ is confirmed. Consequently, the existence of a periodic orbit of the Poincaré return map, and hence of the original continuous-time system, is rigorously verified. This type of interval Newton verification is closely related to rigorous computational techniques used for periodic orbits and periodic windows in chaotic systems [3].

In addition, the derivative of the full return map along the verified cycle is enclosed using interval arithmetic. This makes it possible to study the stability of the verified orbit. In particular, instability can be confirmed when the interval enclosure of a multiplier of P^p is separated from the unit circle.

Expected results

The main output of the computation is a structured list of periodic orbits for periods

$$p = 1, 2, \dots, 20.$$

For each period, the results will be summarized in a table containing the number of Newton-refined periodic orbits, the number of orbits verified by the interval Newton method, and the number of cases for which the verification was not completed with the current settings.

The obtained periodic-orbit counts may also be used to estimate the growth rate of periodic objects. If P_p denotes the number of periodic

orbits with minimal period p , then the number of fixed points of P^n can be reconstructed from the numbers of orbits with periods dividing n . These values can be used to compute finite-period estimates of the topological entropy of the Poincaré return map, following the approach based on periodic-orbit growth [2].

The presented approach therefore combines trajectory monitoring, Newton refinement and interval verification in order to obtain reliable information about periodic structures in a chaotic memristive circuit.

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References

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