

Information fusion enhanced state estimation for a class of nonlinear dynamical systems with unmodeled nonlinearities

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Introduction

State estimation of dynamical systems is a cornerstone of modern control applications. State of the art techniques except data-driven approaches, typically rely on accurate system models to ensure reliable performance. In particular, methods such as observer-based state estimation [4, 5] and predictor-corrector frameworks [3] inherently depend on a known model structure to bound the system's state evolution within guaranteed sets. However, in real-world applications, physical phenomena such as friction, wear, or hysteresis effects are inherently difficult to capture with precise analytical models. We present a state estimation approach for a class of nonlinear dynamical systems, where the structure of the nonlinearity is not required to be known a priori. In contrast to unknown input estimation, this method focuses on identifying state-dependent characteristics. The proposed method is based on information fusion and combines multiple interval observers that incorporate the nonlinearity in the state equations in different ways. Additionally, these interval observers are combined with contractors

[1, 6] to improve the estimation accuracy. The core of the method is the identification of the nonlinearity by intersecting estimated interval boxes [2]. The knowledge of this identified characteristic is used to improve the estimation accuracy of the interval observers. Based on this method, interval observers can be designed that remain effective even in the presence of unmodeled nonlinear dynamics, thereby enhancing the applicability and reliability of state estimation in practical control scenarios.

Main results

As an example, we investigate the state estimation of a mass-spring-damper system with a nonlinear spring characteristic as shown in Fig. 1. The dynamic behavior of the system can be represented by

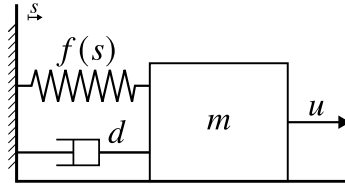


Figure 1: Mass-spring-damper system with a nonlinear spring characteristic $f(s) = -s - s^3$, where s is the spring elongation.

$$\dot{x}_1 = x_2 \quad , \quad (1)$$

$$\dot{x}_2 = f(x_1) - \frac{d}{m}x_2 + \frac{1}{m}u \quad , \quad (2)$$

with $x_1 = s$ and $x_2 = \dot{s}$. The structure of the spring characteristic $f(x_1)$ is not known beforehand. To incorporate this characteristic in the interval observers, it is modeled based on a quasi-linear representation, e.g. $f(x_1) = p(x_1)x_1$, $p(x_1) \in [P]$, where $[P]$ is an interval extension of a state-dependent model coefficient with known bounds. Fig. 2 presents a comparison between the estimation results obtained using the proposed method and those derived from a baseline approach

that relies solely on the quasi-linear representation without information fusion. The estimation accuracy can be enhanced significantly by applying the proposed method.

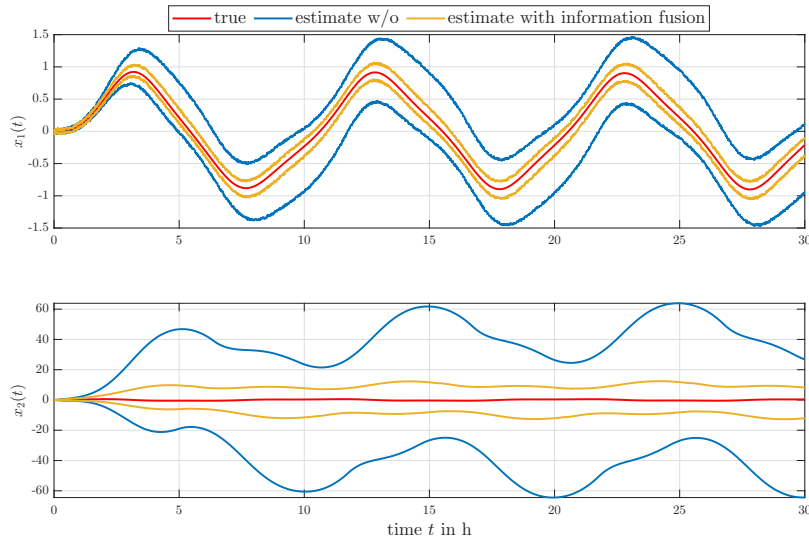


Figure 2: State estimation for the mass-spring-damper system with and without the proposed method.

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