

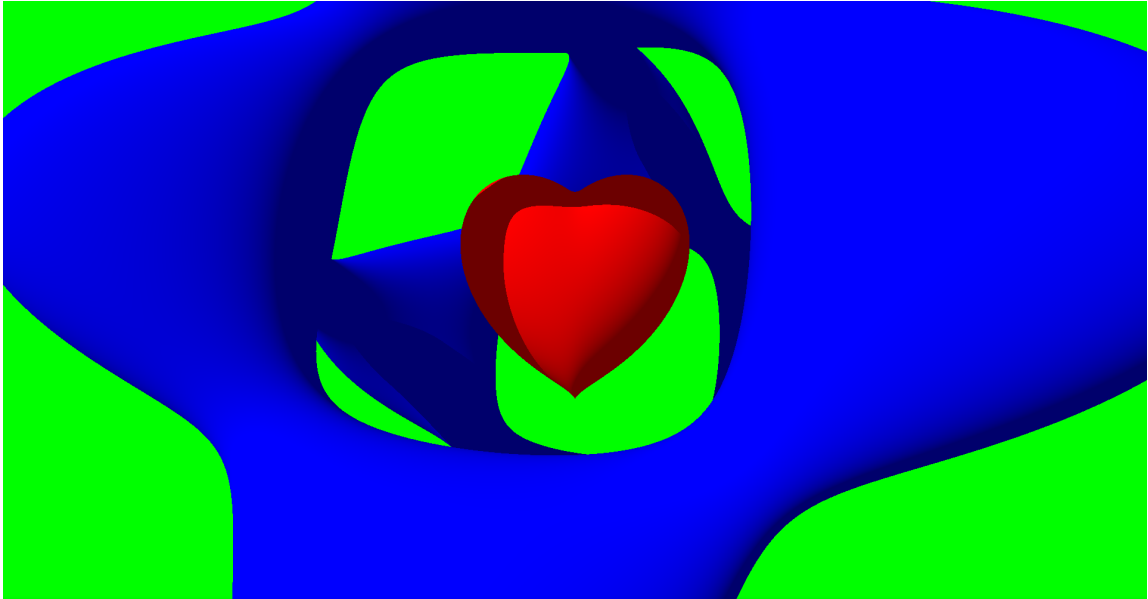
Interval Methods in Surface Intersection Algorithms

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Introduction



An implicit surface is a surface defined by an equation $F(\vec{x}) = 0$. Finding these is useful for various applications, including but not limited to function plotting, rendering or constructive solid geometry.

The surface detection problem is modeled using ray-surface intersections. Let $\vec{r}_o$ be the ray origin and $\vec{r}_d$ the ray direction. The ray is defined as

$$\vec{r}(t) = \vec{r}_o + t\vec{r}_d, \quad t \geq 0 \quad (1)$$

To find a ray-surface intersection we must find the smallest τ for which

$$F(\vec{r}(\tau)) = 0 \quad (2)$$

Traditional point-based algorithms for finding intersections with implicit surfaces are highly error-prone, leading to both under- and over-stepping. By utilizing interval methods, we can provide fast and rigorous ways to reliably locate these surfaces.

Used Methods

We focused on analyzing the traversal[1] algorithm. We implemented and tested

- the algorithm provided in the traversal[1] paper
- an integer based version
- a stack based version
- a newton based version of all of the above

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Used Functions

$$\text{Sphere}(x, y, z) = \sqrt{x^2 + y^2 + z^2} - r$$

$$\begin{aligned} \text{BarthSextic}(x, y, z) = & 4 (\phi^2 x^2 - y^2) (\phi^2 y^2 - z^2) (\phi^2 z^2 - x^2) \\ & - (1 + 2\phi)(x^2 + y^2 + z^2 - 1)^2 \end{aligned}$$

$$\begin{aligned} \text{BarthDecic}(x, y, z) = & 8(\\ & (x^2 - \phi^4 y^2) \\ & (y^2 - \phi^4 z^2) \\ & (z^2 - \phi^4 x^2) \\ & (x^4 + y^4 + z^4 - 2x^2 y^2 - 2x^2 z^2 - 2y^2 z^2)) \\ & + (3 + 5\phi)(x^2 + y^2 + z^2 - w^2)^2 (x^2 + y^2 + z^2 - (2 - \phi)w^2)^2 w^2 \end{aligned}$$

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Main Results

An integer based version turned out to be the fastest of them all. Unfortunately, despite higher theoretical convergence rate, the Newton based algorithm introduced a big overhead due to need of computing the derivative at each step. Further investigation is needed to test if using other methods (e.g. Krawczyk method) would be better. Here are some of the results achieved on a PC with GTX 1060 6GB with $\epsilon = 2^{-12}$ and $d_{\max} = 10$

Table 1: Methods Performance

Method	Sphere	BarthSextic	BarthDecic
Integer-based	0.37ms	6ms	21ms
Original	0.38ms	7ms	26ms
Stack-Based	0.40ms	8ms	29ms
Newton&Integer-based	0.52ms	18ms	97ms
Newton&Original	0.52ms	21ms	122ms
Newton&Stack-Based	0.72ms	55ms	203ms

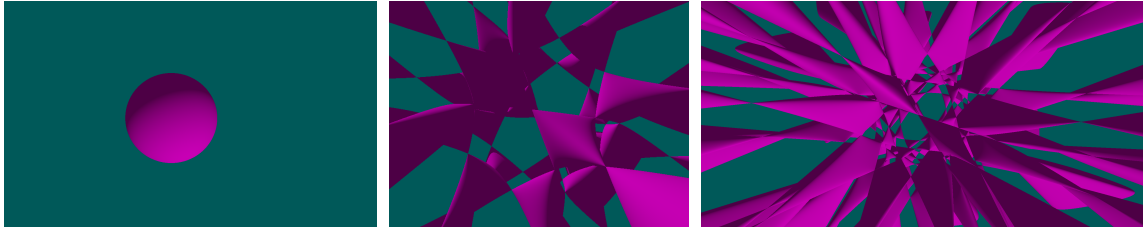


Figure 1: Sphere, BarthSextic and BarthDecic

References

- [1] A. Knoll, Y. Hijazi, A. Kensler, M. Schott, C. Hansen and H. Hagen *Fast Ray Tracing of Arbitrary Implicit Surfaces with Interval and Affine Arithmetic*. *Computer Graphics Forum*, vol. 28, no. 1, pp. 26-40, 2009.